

Performance Assessment Task
Time to Get Clean Grade 3
This task challenges a student to use understanding of time to convert between hours and minutes to make comparisons or combine units of time. A student must demonstrate an understanding of fractions to find parts of an hour. A student must be able to combine units of time in minutes and make comparisons. A student must be able to make sense of different time intervals to find elapsed time.
Common Core State Standards Math - Content Standards
<p><u>Measurement and Data</u></p> <p>Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g. by representing the problem on a number line diagram.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p>
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2008	8	4	50%

Time to Get Clean

This problem gives you the chance to:

- work with a table of activities and times
-

Here is a list showing what happens in Megan and Carl's bathroom every morning.

$1 \text{ hour} = 60 \text{ minutes}$



<i>Person</i>		<i>Time taken</i>
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

1. Who spends the most time in the bathroom? _____
2. Who spends the shortest time in the bathroom? _____
3. How long do Dad and Grandpa spend in the bathroom, in all? _____
Show how you figured this out.
4. How much longer does Megan spend in the bathroom than Carl? _____
5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? _____
Show how you figured this out.

Time to Get Clean	Rubric	
The core elements of performance required by this task are: • work with a table of activities and times Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: Dad	1	1
2. Gives correct answer: Carl	1	1
3. Gives correct answer: 1 hour 25 minutes accept 85 minutes Shows correct work such as: 50 minutes + 35 minutes	1 1	2
4. Gives correct answer: 10 minutes	1	1
5. Gives correct answer: 9 a.m. Shows correct work such as: $30 + 20 + 45 + 50 + 35$ $= 180 \div 60 = 3 \text{ hours}$ <i>Partial credit</i> One error	1 2 (1)	3
Total Points		8

Time to Get Clean

Work the problem and look at the rubric. What are the key mathematics a student needs to understand in order to be successful on this task? _____

Now look at student work on part 1 and 2:

How many of your students could:

- Identify Dad as taking the longest: _____
- Confused mom ($\frac{3}{4}$) as longer than 50 minutes: _____
- Thought $\frac{1}{2}$ hour (Meagan) was the longest:

For the shortest time, how many students picked:

- Carl (20 minutes) _____
- Megan ($\frac{1}{2}$ hour) _____
- Grandpa (35 minutes) _____

Now look a student work for part 4, how much longer did Megan spend in the bathroom than Carl.

How many of your students put:

10 minutes	30 min. or $\frac{1}{2}$ hour	40 minutes	1 hour	20 minutes	Other

Which errors were caused by not understanding $\frac{3}{4}$ of an hour?

Which errors were caused by using an incorrect operation?

What other misconceptions came into play in student thinking?

Now look at student work in part 5.

- How many of your students could change $\frac{1}{2}$ hr. to 30 minutes? _____
- How many could change $\frac{3}{4}$ hr. to 45 minutes?
- Make a list of other values for $\frac{3}{4}$ of an hour:

- When adding, were students able to add minutes? _____
- How many attempted to convert their answer into hours and minutes? _____
- How many tried to put the minutes into time notation with a colon? _____
- What were some examples of strange things students did with fractions?
- How many of your students had some conception about elapsed time versus just an answer in hours and minutes? _____

What kinds of experiences do your students need with understanding common units of measure?

With learning the logic of conversions? What other ideas do they need more practice with?

Looking at Student Work on Time to Get Clean

Student A has the habit of mind to label all the work and keep track of the all the calculations. The student is comfortable converting fractions of an hour to minutes. The student knows how to change minutes into hours and find elapsed time versus amount of time.

Student A

Person		Time taken
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

- Who spends the most time in the bathroom? Dad ✓
- Who spends the shortest time in the bathroom? Carl ✓
- How long do Dad and Grandpa spend in the bathroom, in all? 1 hour & 25 mins. ✓
Show how you figured this out.

$$\begin{array}{r}
 50 = \text{dad} \\
 + 35 = \text{grandpa} \\
 \hline
 85 = \text{altogether} \\
 \text{min.}
 \end{array}$$

$$85 \text{ min.} = 1 \text{ hour and } 25 \text{ mins.}$$
- How much longer does Megan spend in the bathroom than Carl? 10 more ✓ mins.
- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 9:00 am ✓
Show how you figured this out.

$$\begin{array}{r}
 180 = 3 \text{ hours} \\
 30 \text{ min.} = \text{Megan} \\
 20 \text{ min.} = \text{Carl} \\
 + 45 \text{ min.} = \text{Mom} \\
 50 \text{ min.} = \text{Dad} \\
 35 \text{ min.} = \text{grandpa} \\
 \hline
 180
 \end{array}$$

$$\begin{array}{r}
 6:00 \text{ am} \\
 + 3 \text{ hours} \\
 \hline
 9:00 \text{ am}
 \end{array}$$

8

Student B is able to add time in part 3 and has an interesting strategy for converting minutes to hours. Unfortunately the student doesn't use proper notation for the answer, writing a time of day instead of an answer in hours and minutes. In part 5 the student is correctly able to change all the fractions of an hour into minutes, but doesn't understand elapsed time. The student tries to add minutes to a time of day. This is probably caused or related to the misunderstanding of notation, which would identify them as separate types of objects: time and hours.

Student B

1. Who spends the most time in the bathroom? Dad ✓

2. Who spends the shortest time in the bathroom? Carl ✓

3. How long do Dad and Grandpa spend in the bathroom, in all? 1:25 x ✓
 Show how you figured this out.

$$\begin{array}{r} 50 \\ + 35 \\ \hline 85 \end{array}$$

$$60 = 1$$

$$120 = 2$$

$$\begin{array}{r} 80 \\ + 20 \\ \hline 100 \\ + 5 \\ \hline 105 \end{array}$$

$$\begin{array}{r} 1:20 \\ + 5 \\ \hline 1:25 \end{array}$$

4. How much longer does Megan spend in the bathroom than Carl? 10 x ✓

$$30 - 20 = 10$$

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 7:80 x ✓
 Show how you figured this out.

$$\begin{array}{r} 6:00 \\ + 30 \\ + 20 \\ + 45 \\ \hline 6:95 \end{array}$$

$$\begin{array}{r} 6:45 \\ + 50 \\ + 35 \\ \hline 7:80 \end{array}$$

8

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Third Grade - Time to Get Clean

Student C makes the common error of confusing $\frac{3}{4}$ of an hour with 40 minutes. The student is able to do all the thinking needed for conversions between hours and minutes and finding elapsed time.

Student C

Person		Time taken
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour 30min
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour 40min
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

- Who spends the most time in the bathroom? Dad ✓
- Who spends the shortest time in the bathroom? Carl ✓
- How long do Dad and Grandpa spend in the bathroom, in all? 1hr. 25min ✓
Show how you figured this out.

$$\begin{array}{r} 50 \\ +35 \\ \hline 85 \end{array} \quad \begin{array}{r} 60 \\ +25 \\ \hline 85 \end{array} \quad \begin{array}{r} 1 \\ 1 \\ 1 \\ 1 \end{array}$$

- How much longer does Megan spend in the bathroom than Carl? 10 min ✓

$$\begin{array}{r} 30 \\ -20 \\ \hline 10 \end{array}$$

- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 8:55 a.m. X 0
Show how you figured this out.

$$\begin{array}{r} 2 \text{ hr. } 55 \text{ min.} \\ 6 \\ +2 \\ \hline 8 \end{array} \quad \begin{array}{r} 30 \text{ min.} \\ 20 \text{ min.} \\ 40 \text{ min.} \times \\ 50 \text{ min.} \\ +35 \text{ min.} \\ \hline 175 \text{ min.} \end{array} \quad \begin{array}{r} 60 \\ +60 \\ \hline 120 \end{array} \quad \begin{array}{r} 175 \\ +120 \\ \hline 055 \end{array} \quad \begin{array}{r} 0 \\ 0(1) \\ 5 \\ 6 \end{array}$$

8

Student D is able to convert fractions of an hour to minutes and correctly add the minutes. Notice in part 5 the student is able to think about the 180 as three groups of 60, but doesn't make the connection to 3 hours that elapse from 6:00 a.m. *What might be the next steps with this student?*

Student D

Person		Time taken
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

1. Who spends the most time in the bathroom?

Dad ✓

1

2. Who spends the shortest time in the bathroom?

Carl ✓

1

3. How long do Dad and Grandpa spend in the bathroom, in all?

85 minutes ✓

1

Show how you figured this out.

$$\begin{array}{r} 50 \\ + 35 \\ \hline 85 \end{array} \quad \checkmark$$

1

4. How much longer does Megan spend in the bathroom than Carl?

10 more minutes ✓

1

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free?

7:00 am ✗

0

Show how you figured this out.

$$\begin{array}{r} 30 \\ 20 \\ 45 \\ 35 \\ + 50 \\ \hline 180 \end{array} \quad \checkmark$$

$$\begin{array}{r} 160 \\ 60 \\ + 60 \\ \hline 180 \end{array}$$

2

8 (7)

Student E changes $\frac{3}{4}$ of an hour into 4 hours. Student E thinks about $\frac{1}{2}$ as thirty minutes, but thinks a one needs to go in front for the fraction. Notice the student uses time notation instead of hours and minutes notation in part 5. So while the amounts give an answer larger than six hours, the notation gives the illusion only 35 minutes have elapsed. The student also does not understand that because hours and minutes are not related by a multiple of 10, the numbers can't carry between minutes and hours. Looking at this work raises the question about how notation effects the thinking. *Using time notation eliminates the possibility of thinking about conversions between hours and minutes. It also gives the appearance of calculating a time instead of an addition of hours and minutes, so on the surface it looks like an elapsed time. It is a fundamental misunderstanding of the mathematical ideas. How could you design a class discussion to help surface the misconceptions in this work?*

Student E

Person		Time taken
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour 1:30
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

- Who spends the most time in the bathroom? Dad ✓
- Who spends the shortest time in the bathroom? Carl ✓
- How long do Dad and Grandpa spend in the bathroom, in all? 85 min ✓
Show how you figured this out.
 $50 + 35 = 85$
- How much longer does Megan spend in the bathroom than Carl? 10 min ✓
 $1:30 - 20 = 1:10$
- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 6:35 x 0
Show how you figured this out.

$$\begin{array}{r}
 1:30 \\
 4:20 \\
 \hline
 5:50 \\
 35 \\
 \hline
 6:35
 \end{array}$$

0
5
8

Student F also has troubles thinking about how to convert fractions of an hour to minutes. For three fourths of an hour the student uses 340 minutes. The student has some idea about 1/2 hour being 30 minutes, but for consistency makes it 130 minutes. Notice that the student uses 90 minutes for 1/2 hour in part 4. *Can you think about why the student may have had two different values for 1/2 hour? Is there some consistency in the thinking?* Notice that in part 5 the student converts total minutes into time notation 550 minutes equals 5:50 a.m. Then the student tries to add to different times together. *What are some of the implications in papers F and E about what it takes to understand time measurement?*

Student F

1. Who spends the most time in the bathroom? mom X

2. Who spends the shortest time in the bathroom? Carl ✓

3. How long do Dad and Grandpa spend in the bathroom, in all? 85
 Show how you figured this out.

$$\begin{array}{r} 50 \text{ min} \\ + 35 \text{ min} \\ \hline 85 \text{ min} \end{array}$$

4. How much longer does Megan spend in the bathroom than Carl? 70 min

$$90 \text{ min} - 20 \text{ min} = 70 \text{ min}$$

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 7:55 am
 Show how you figured this out.

$$\begin{array}{r} 130 \\ + 20 \\ \hline 150 \end{array} \quad \begin{array}{r} 130 \\ + 340 \\ \hline 470 \end{array} \quad \begin{array}{r} 470 \\ + 50 \\ \hline 520 \end{array} \quad \begin{array}{r} 520 \\ + 35 \\ \hline 555 \end{array} \quad \begin{array}{r} 6:00 \text{ am} \\ + 5:55 \\ \hline 11:55 \text{ am} \end{array}$$

Student G doesn't understand how to change 1/2 hour into minutes. Notice the strategy for subtracting fractions. *Can you explain what the student is doing in part 4?* Student G doesn't make any connection between the information in the table and finding out when the bathroom will be free.

Student G

1. Who spends the most time in the bathroom? Dad ✓

2. Who spends the shortest time in the bathroom? Megan ✗

3. How long do Dad and Grandpa spend in the bathroom, in all? 85 min. ✓
Show how you figured this out.

$$\begin{array}{r} 50 \\ + 35 \\ \hline 85 \end{array}$$

4. How much longer does Megan spend in the bathroom than Carl? 17 ✗

$$\begin{array}{r} 1 \cancel{2} 0^{10} \\ - \quad \quad \quad \frac{1}{2} \\ \hline \end{array}$$

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 7 a.m. ✗
Show how you figured this out.

$$\begin{array}{r} 6 \text{ a.m.} \\ + \quad \quad \quad \frac{1}{2} \text{ h.} \\ \hline 7 \text{ a.m.} \end{array}$$

Student H takes the numerator of the fraction as an hour: $\frac{1}{2}$ hour = 1 hour and $\frac{3}{4}$ hour = 3 hours. The student then adds correctly and finds the correct elapsed time for those incorrect values. The student doesn't understand the correct operation for finding out the comparison in part 4.

Student H

Person		Time taken
Megan	Showers, washes and dries hair, brushes teeth	$\frac{1}{2}$ hour
Carl	Showers, brushes teeth	20 minutes
Mom	Takes a bath, brushes teeth	$\frac{3}{4}$ hour
Dad	Showers, shaves, brushes teeth	50 minutes
Grandpa	Showers, shaves	35 minutes

1. Who spends the most time in the bathroom?

Mom ~~X~~ 0

2. Who spends the shortest time in the bathroom?

Carl ✓ 1

3. How long do Dad and Grandpa spend in the bathroom, in all?
Show how you figured this out.

1 25
hour min

~~X~~

4. How much longer does Megan spend in the bathroom than Carl?

1 100
hour min

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free?
Show how you figured this out.

~~11:45~~

Mom takes 3 hours
Carl takes 20 mins
Megan takes 1 hour
Grandpa takes 35 mins
Dad takes 50 mins

8

Student I is able to find a correct solution to part 3, but has some troubling ideas about place value. *What types of experiences do you think Student I needs?* In part 1 and 5 the student gives multiple answers. *Do you think the student doesn't understand that the mathematics should give one solution or is it just having no strategies for making sense of the information?*

Student I

Grandpa	Showers, shaves	35 minutes
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- Who spends the most time in the bathroom? Mom ($\frac{3}{4}$) or land Megan ($\frac{1}{2}$ h)
- Who spends the shortest time in the bathroom? Carl?
- How long do Dad and Grandpa spend in the bathroom, in all?
Show how you figured this out. 85
 $5 + 3 = 8$ add 0 = 80, add 5, 85! -
- How much longer does Megan spend in the bathroom than Carl? 10 min
- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free?
Show how you figured this out. 2, 3 or 4 hours
I guessed??

Look at the work of Student J. *How does the student convert from fractions of an hour to minutes?* *How does the student's understanding of measurement and conversions lead to incorrect calculations and problems with thinking about elapsed time?*

Student J

- Who spends the most time in the bathroom? Mom
- Who spends the shortest time in the bathroom? Carl
- How long do Dad and Grandpa spend in the bathroom, in all?
Show how you figured this out. 85
$$\begin{array}{r} 50 \\ + 35 \\ \hline 85 \end{array}$$
- How much longer does Megan spend in the bathroom than Carl? 40
- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free?
Show how you figured this out. 1:11
$$\begin{array}{r} 50 \\ + 35 \\ + 20 \\ \hline 105 \end{array}$$

B.O.D

Page 5 Third Grade - Time to Get Clean

Student K doesn't understand the mathematics in the problem. The student answers part 5 with practical knowledge rather than mathematics, but gives an elapsed time of 11 hours. *What questions might you ask student K to check for any understanding about minutes and hours or time in general? What would you like to know?*

Student K

Grandpa	Showers, shaves	35 minutes
---------	-----------------	------------

- Who spends the most time in the bathroom? Megan ✓ x ⊕
- Who spends the shortest time in the bathroom? Carl ✓ x ⊕
- How long do Dad and Grandpa spend in the bathroom, in all?
Show how you figured this out. 25 minutes x ⊕
- How much longer does Megan spend in the bathroom than Carl? 70 x ⊕
- The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free?
Show how you figured this out. 5:00 x ⊕

TO mush people take
showers. (1) 8

Student Task	Work with a table of activities and times. Work with common fractions and add and convert between measures of time.
Core Idea 4 Geometry and Measurement	Understand and use appropriate techniques to determine measurements. <ul style="list-style-type: none"> • Choose appropriate units and tools for particular tasks and use these units to measure time.
Core Idea 1 Number Properties	<ul style="list-style-type: none"> • Develop an understanding of fractions as a part of unit whole, as part of a collection, and as a location on a number line.

Mathematics of the task:

- Making conversions between fractions of an hour and minutes
- Identifying largest and smallest amounts of time and knowing that units need to be the same to make a comparison
- Adding minutes
- Converting minutes to hours and minutes
- Understanding how to take hours to find elapsed time

Based on teacher observations, this is what third graders knew and were able to do:

- Add minutes to find time 2 people were in the bathroom
- Find the shortest amount of time

Areas of difficulty for third graders:

- Converting $\frac{1}{2}$ hour and $\frac{3}{4}$ of an hour into minutes
- Understanding the difference between hours and minutes (a quantity of time) and time of day notation 5:30 is either a time in the morning or afternoon (different from 5 hours and thirty minutes).
- Finding elapsed time
- Knowing that you can't add time the quantity to time of day

Strategies used by successful students:

- Used groups of 60 and repeated subtraction to convert minutes to hours
- Could convert or "just knew" how to change fractions of an hour to minutes
- Found elapsed times in increments by thinking about a clock face (6:00 + 30 minutes = 6:30)

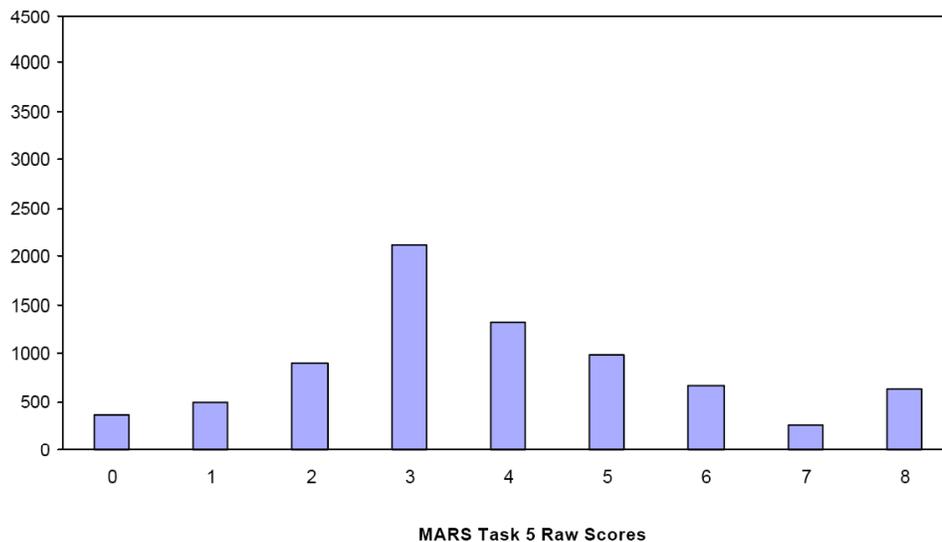
Task 5 - Time To Clean

Mean: 3.84 StdDev: 2.03

Table 19: Frequency Distribution of MARS Test Task 5, Grade 3

Task 5 Scores	Student Count	% at or below	% at or above
0	356	4.6%	100.0%
1	498	11.0%	95.4%
2	908	22.7%	89.0%
3	2127	50.2%	77.3%
4	1317	67.2%	49.8%
5	990	80.0%	32.8%
6	667	88.6%	20.0%
7	256	91.9%	11.4%
8	630	100.0%	8.1%

Figure 28: Bar Graph of MARS Test Task 5 Raw Scores, Grade 3



*The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.*

Most students, 89%, could read the table and find the total minutes two people were in the bathroom. Many students, about 77%, could also identify the person who spent the least amount of time in the bathroom. Almost half the students, 49%, could find the person with the least time, and the times of two people together, convert 1/2 hour to 30 minutes, and use comparison subtraction to find out how much longer one person was in the bathroom than someone else. Some students, about 33%, could also identify the person who spent the most time in the bathroom. About 8% of the students could meet all the demands of the task, including converting 3/4 of an hour to minutes, finding the total time 5 people spent in the bathroom, convert minutes to hours and minutes, and find elapsed time. Almost 5% of the students scored no points on the task. 70% of the students with this score attempted the task.

Time to Get Clean

Points	Understandings	Misunderstandings
0	72% of the students with this score attempted the task.	Students had trouble adding the minutes in part 3. Some made calculation errors (87, 80). Some students had difficulty with the conversion (1 hr. 35 minutes, 1 hours 15 minutes). Some students used time notation (7:25, 1:35).
2	Students could find values in the table and add minutes (like units).	Students had difficulty determining the smallest time in the table. 9% of the students thought 1/2 hr. was smallest. 8% thought 35 minutes was smallest.
3	Students could identify the smallest amount of time and add minutes for two people.	Students had difficulty comparing two times (how much longer). 18% tried to add. 8% just put 30 min. (Megan's time) rather than subtracting. 3% put Carl's time (20 minutes). At least 22% made errors in thinking about changing 1/2 hour to minutes.
4	Students could convert 1/2 hour to 30 minutes. Students could identify the smallest amount of time, add minutes, and compare times by subtraction.	Students had difficulty identifying the largest amount of time. Students did not know the value for 3/4 of an hour. 45% of the students thought 3/4 of an hour was the most (Mom). 13% thought 1/2 hour was the most (Megan).
5	Students could recognize the largest and smallest amounts of time, add minutes, and do comparison subtraction. 51% showed evidence of 1/2 hour = 30 min. 35% showed evidence that 3/4 hour = 45 min.	Students did not know how to find elapsed time. Most common wrong amounts for 1/2 hour were 1/2, 1:30 or 130, and 1 hour. Most common wrong amounts for 3/4 of an hour were 3/4, 40, 15, and 50 min. 38% of the students showed no strategy for their answers to part 5. 11% treated total minutes as a time, e.g. 220 minutes = 2:20. 6% just left the total minutes and did not convert or find elapsed time. 4% converted minutes to hours and minutes, but did not find elapsed time. 4% added time to time, e.g. 6:00 + 2:20 = 8:20.
8	Students could work flexibly with units of time, knew 1/2 hour and 3/4 hour in minutes, and convert total minutes to hours and minutes. Students had strategies for thinking about elapsed time.	

Implications for Instruction

Students need to recognize common fractions of an hour, such as $\frac{1}{2}$ or $\frac{3}{4}$, and be able to convert them into minutes. Students should be able to compare time by subtraction and identify most and least amounts of time from a list. Students should have strategies for converting between hours and minutes and hours, or vice versa. Students should have experiences with different types of measurement situations and be able to generalize about making conversions among units. Students need to also be familiar with the notational formats of measurement, going from larger to progressively smaller units.

The idea of elapsed time is complicated for students. Many students confuse time of day with time spent on an activity. While students may know how to do conversions from minutes to hours when dealt with alone, they struggle with the logic of converting minutes to hours in the context of time of day.

Ideas for Action Research – Re-engagement on Elapsed Time and Model Design

One useful strategy when student work does not meet your expectations is to use sample work to promote deeper thinking about the mathematical issues in the task. In planning for re-engagement it is important to think about what is the story of the task, what are the common errors and what are the mathematical ideas that students need to think about more deeply. Then look through student work to pick key pieces of student work to use to pose questions for class discussion. Often students will need to have time to rework part of the task or engage in a pair/share discussion before they are ready to discuss the issue with the whole class. This reworking of the mathematics with a new eye or new perspective is the key to this strategy.

To plan a follow-up lesson using this task, pick some interesting pieces of student work that will help students confront and grapple with some of the major misconceptions. Make the misconceptions explicit and up for public debate.

During the discussion, it is important for students to notice and point out the errors in thinking.

In this task the story is the difficulties students had with making conversions and then how to deal with conversions in the context of elapsed time. It is often beneficial to spend considerable time on the simpler part of task to allow students to talk their way into understanding, before grappling with the core issue. In this problem, the teacher might start with the issue of how long did it take for Dad and Grandpa to get clean. The teacher might ask:

I saw this work on someone's paper. Can you figure out what the student was doing?

$$50 + 35 = 85$$

When students have reached an agreement about units and purpose of this first calculation, follow it with something a bit more puzzling.

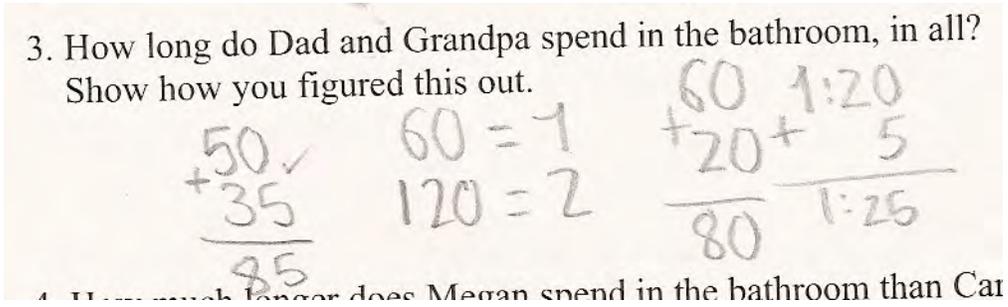
The same student then wrote: $60 + 25 = 85$

What was the student thinking? What was the purpose of this calculation? What do you think the student might do next?

Hopefully, students will see that the student has changed the total time into hours and minutes. Now have students confront the idea of notation for time with this prompt:

Another student did this work. Can you figure out what the student was thinking?

3. How long do Dad and Grandpa spend in the bathroom, in all?
Show how you figured this out.



4. How much longer does Megan spend in the bathroom than Car

What do the two 120's represent?

What error has the student made?

Now it is time to tackle the idea of elapsed time. Here instead of doing a re-engagement using student work, the teacher might do a re-engagement to help the students try on a new strategy. The teacher might first ask students to agree on the length of time in minutes for 1/2 hour and 3/4 hour, so everyone is starting from a level playing field. The idea here is to focus on the mathematics of elapsed time.

Sometimes in working with problem solving, it is important to ask clarifying questions to make sure everyone understands the prompt. The teacher might ask students to explain what it means to have everyone be finished getting clean. Does this mean everyone gets clean at once? When everyone has the idea of consecutive turns, the teacher might ask students to try to make a model of picture to help them think about the problem.

I noticed that many students had difficulty finding out what time everyone would be finished. I am wondering if you can make a drawing or model to go with your calculations that will help you figure out when everyone is done in the bathroom.

Now the teacher might use models from the previous papers or models from this current prompt to now have a discussion about elapsed time. Here are some examples of possible work from the toolkit papers:

What is this student thinking? What suggestions would you make to help the student?

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 8:45 a.m.
 Show how you figured this out.

$$50 + 30 + 5 + 30 + 50 = 215$$

or

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 8:35
 Show how you figured this out.

$$6:00 + 30 \text{ minutes} = 6:30 + 20 \text{ minutes} = 6:50$$

$$+ 50 \text{ minutes} = 7:35 + 35 \text{ minutes} = 8:35$$

or

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 7:00 am
 Show how you figured this out.

$$\begin{array}{r} 30 \\ 20 \\ 45 \\ 35 \\ + 50 \\ \hline 180 \end{array}$$

$$\begin{array}{r} 160 \\ 60 \\ + 60 \\ \hline 180 \end{array}$$

or

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 11:55 am
 Show how you figured this out.

$$\begin{array}{r} 130 \\ + 20 \\ \hline 150 \end{array}$$

$$\begin{array}{r} 130 \\ + 340 \\ \hline 470 \end{array}$$

$$\begin{array}{r} 470 \\ + 50 \\ \hline 520 \end{array}$$

$$\begin{array}{r} 520 \\ + 35 \\ \hline 555 \end{array}$$

$$\begin{array}{r} 6:00 \text{ am} \\ + 5:55 \\ \hline 11:55 \text{ am} \end{array}$$

or

5. The first person goes into the bathroom at 6 a.m. and it is in use until everyone has finished getting clean. At what time will the bathroom be free? 6:35
 Show how you figured this out.

$$\begin{array}{r} 1:30 \\ 4:20 \\ 4:00 \\ 50 \\ 35 \\ \hline 6:35 \end{array}$$